

Review

- §1-1 → use reasoning to make predictions.  
(inductive) (conjecture)
- §1-2 → determine if the conjecture is valid (testing)
- §1-3 → find a counterexample (or contradiction)
- §1-4 → proving the conjectures. (mathematical proof  
(deductive reasoning) or a logical argument)

Mid Chapter Review (p34-35)

10.  $(2+3)^2 = 5^2 = 25$  is  $25 > \sqrt{2^2 + 3^2}$  ?

$(5+7)^2 = 12^2 = 144$  is  $144 > \sqrt{5^2 + 7^2}$  ?

$(6+3)^2 = 9^2 = 81$  is  $81 > \sqrt{6^2 + 3^2}$  ?

Let  $x$  be the first <sup>positive</sup> integer  
 $y$  be the second positive integer

Prove:  $(x+y)^2 > x^2 + y^2$

Working with  $(x+y)^2$ :

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x+y)^2 = \underline{x^2 + y^2} + 2xy$$

Since  $x$  and  $y$  are both positive integers  
 $2xy$  will be greater than zero and  $x^2 + y^2 + 2xy$   
will be greater than  $x^2 + y^2$

11. Let  $2x+1$  be any odd integer  
Let  $D$  be the difference between the square of  
the integer and the integer itself.

Prove  
that  
 $D$  is even

$$D = (2x+1)^2 - (2x+1)$$

$$D = 4x^2 + 4x + 1 - 2x - 1$$

$$D = 4x^2 + 2x$$

$$D = 2(2x^2 + x)$$

Since this difference is divisible by 2, then

§1.5 Proofs That Are Not Valid (p36)INVESTIGATE the Math

Moh claimed that  $64 = 65$ ! Watch out for overlap or empty space! (see p36)

Watch out for Proofs that contain errors

- ① Proofs that begin with a false statement (Example 1)
- ② Algebraic Errors (Example 4)
- ③ Division by zero (Example 2)
- ④ Circular Reasoning (Example 3)

Example 1 (p37)

False Statement  $\Rightarrow$  athletes do not compete in both the winter and summer olympics.

Example 2

Bev claims that he can prove  $3 = 4$ .

Bev's Proof:

Suppose that  $a + b = c$

$$4a - 3a \quad 4b - 3b \quad 4c - 3c$$

$$(4a - 3a) + (4b - 3b) = (4c - 3c)$$

$$4a + 4b - 4c = 3a + 3b - 3c \quad (\text{rearrange})$$

$$\frac{4(a + b - c)}{(a + b - c)} = \frac{3(a + b - c)}{(a + b - c)}$$

$$\frac{0}{0} = \frac{0}{0}$$

$$4 = 3$$

$\uparrow$  if  $a + b = c$   
 $a + b - c = 0$

This proof is invalid since you cannot divide by zero.

Example 3

Liz claims that she has proved  $-5 = 5$ .

Liz's Proof: I assumed  $-5 = 5$

$$(-5)^2 = 5^2 \quad (\text{square both sides})$$

$$25 = 25$$

Circular Reasoning -

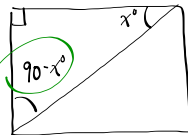
Therefore  $-5 = 5$

an argument that is incorrect because it makes use of the conclusion you are trying to prove.

Another example of circular reasoning:

Darren claims that he can prove that the sum of the interior angles in a triangle is  $180^\circ$ .

He draws a rectangle:



Darren used what he was trying to prove in his argument.

he used what he was trying to prove

$$90^\circ + (90^\circ - x^\circ) + x^\circ = 180^\circ$$

You cannot use what you are trying to prove.

Example 4

Her proof

$x$

Choose any #

$x + 3$

add three

$2x + 6$

double it

$2x + 10$

add 4

mistake

$$\begin{matrix} x+5 \\ \rightarrow 2x+5 \end{matrix}$$

divide by 2

$x + 5$

take away the number you started with

$$\textcircled{5}$$

TO DO

① C4U (p 42)

② p 42 | 3-7, 10

